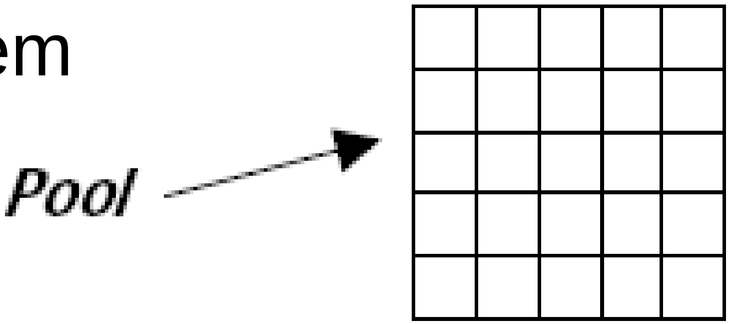
Teaching Through Problem Solving

Session 4

Prom Dress Problem

Gumball Problem



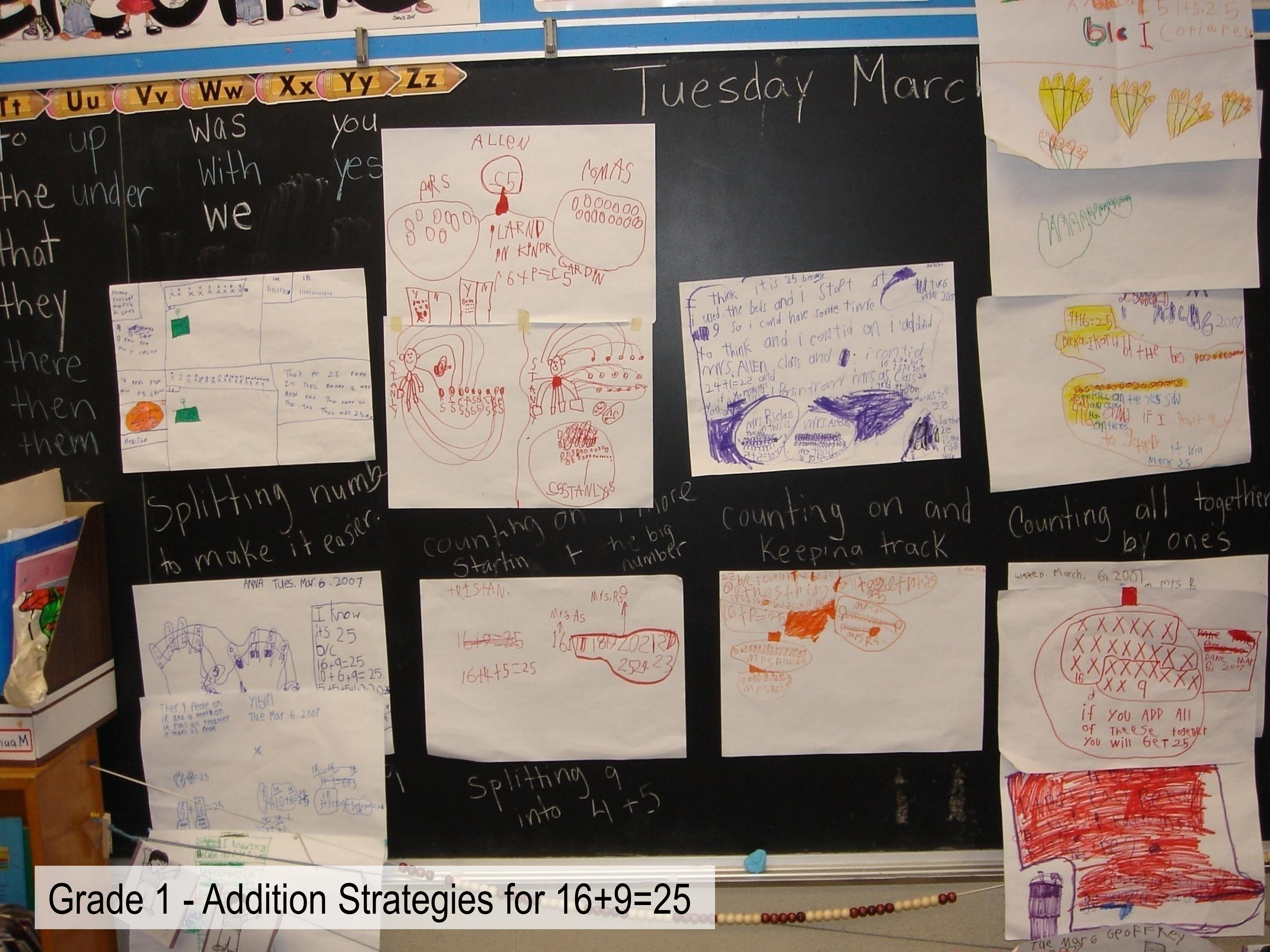
facilitated by
Kathy Kubota-Zarivnij

GAINS
Growing Accessible Interactive Networked Supports

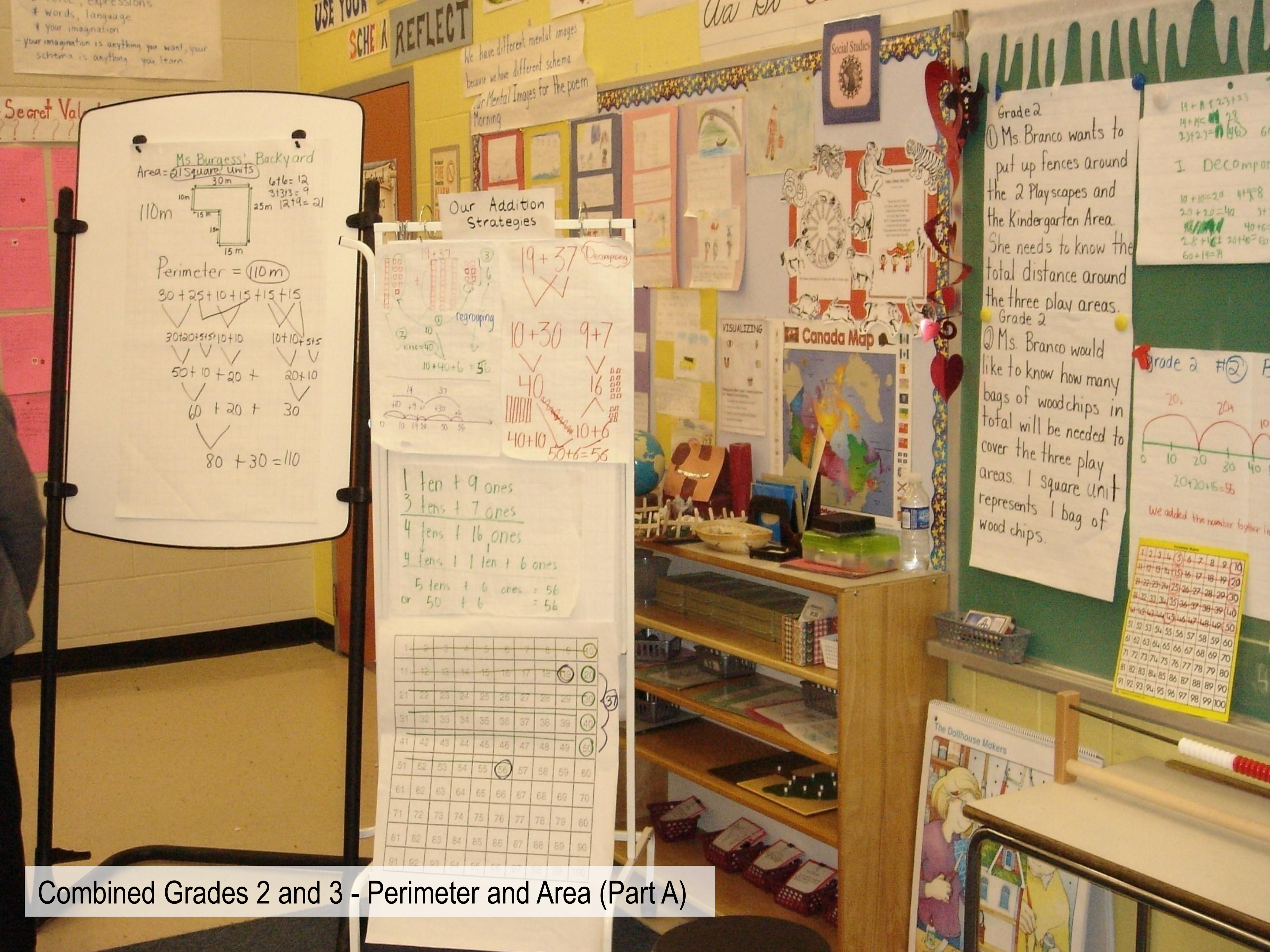
- use curriculum expectations and mathematical processes (i.e., grades 6 to 10) to anticipate a range of solutions to a problem, to understand and describe the mathematics in student solutions, and to judge the appropriateness of problems for teaching/learning
- develop a knowledge package or landscape of clustered curriculum expectations
- use the three-part problem solving lesson design to frame the use of problems for teaching mathematics
- develop strategies and mathematics knowledge for anticipating student responses and understanding students' mathematical thinking
- experience strategies for developing students' mathematical communication through the discourse of a math-talk learning community, teacher recording strategies (blackboard writing – mathematical annotations), and coordination and recording of discussion (bansho)
- vary the structure of the problem for students to practise new learning and to provoke use of some strategies and not others, moving towards strategies that can generalize

Bansho Reinvented for Ontario Classrooms - "Ontario" Bansho

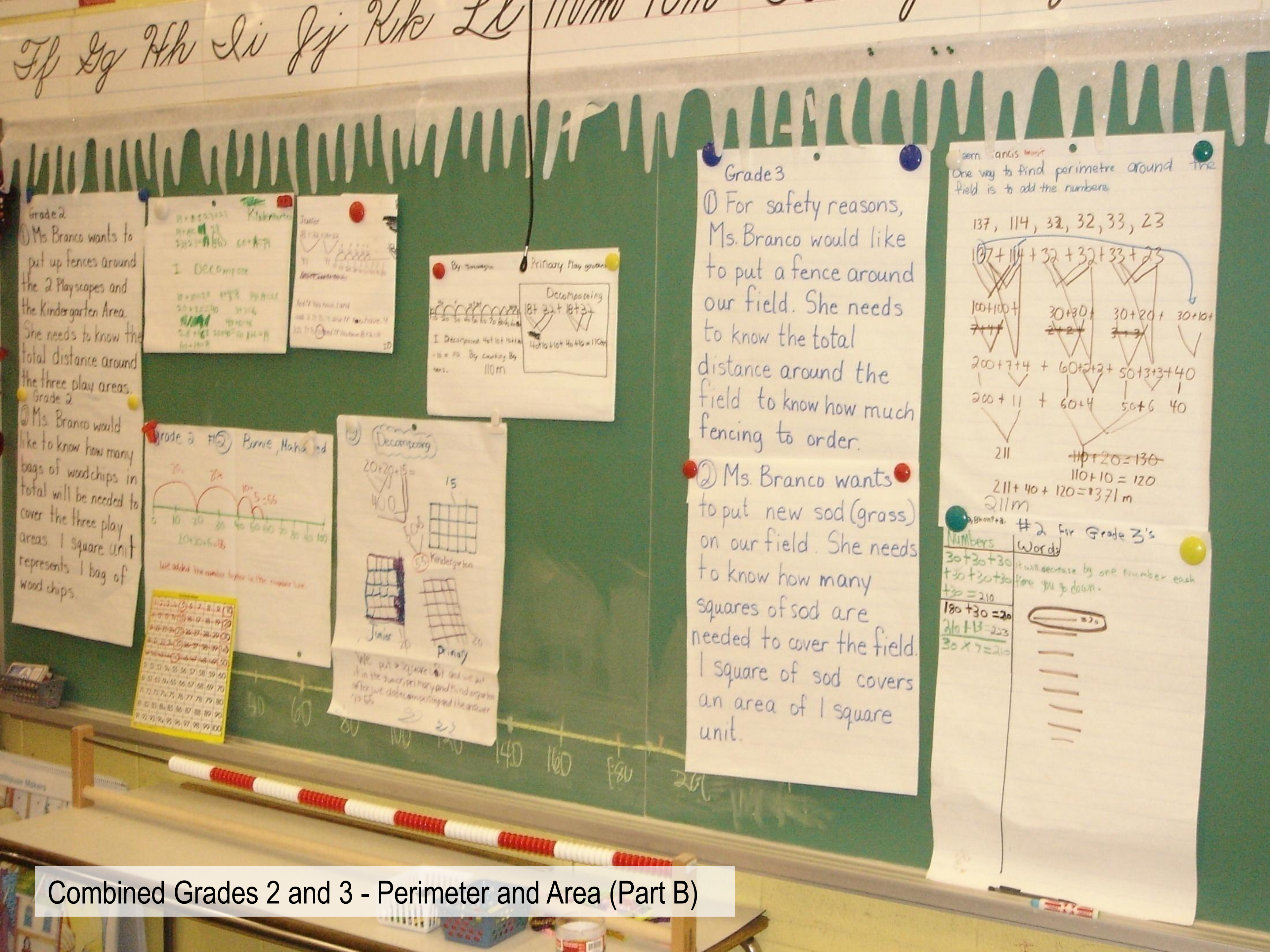
- a **mathematics instructional strategy** that provokes students' mathematical thinking to be explicit when solving problems through the organization and annotation of student work samples and classroom discourse
- a **classroom artefact** produced collectively by students and teacher that publicly displays the mathematical relationship among students' solutions; could be used as a mathematics landscape for learning or as a mathematics anchor chart
- an **assessment for learning strategy** to discern the range and mathematical relationships between ideas, strategies, and models of representation used to solve a problem by students
- a **professional learning strategy** that develops teacher's knowledge of mathematics *for teaching,* to construct a mathematics landscape evoked through the solving of problems



Grade 1 - Addition Strategies for 16+9=25



Combined Grades 2 and 3 - Perimeter and Area (Part A)



Combined Grades 2 and 3 - Perimeter and Area (Part B)

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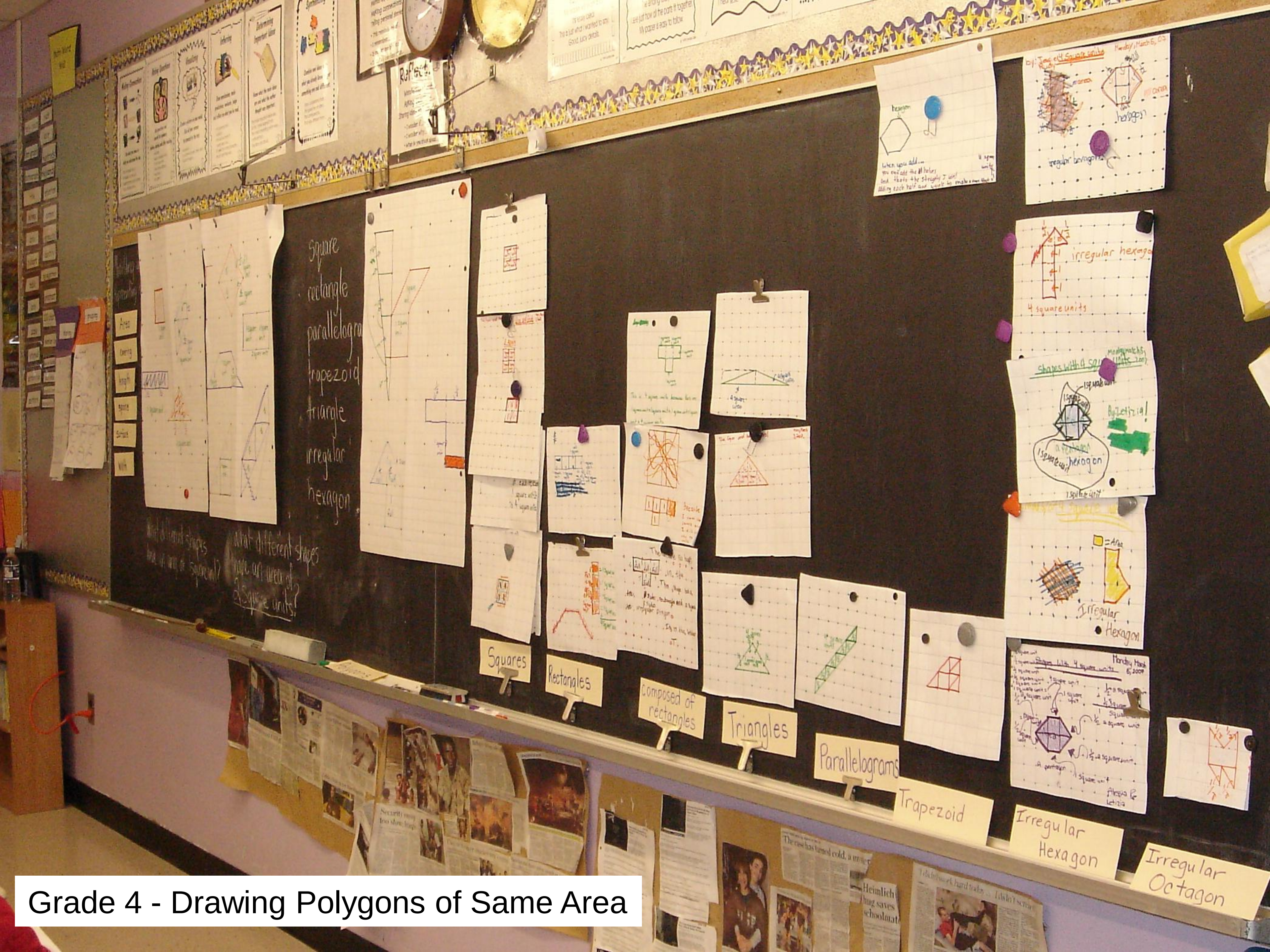

Grade 4 - Drawing Polygons of Same Area

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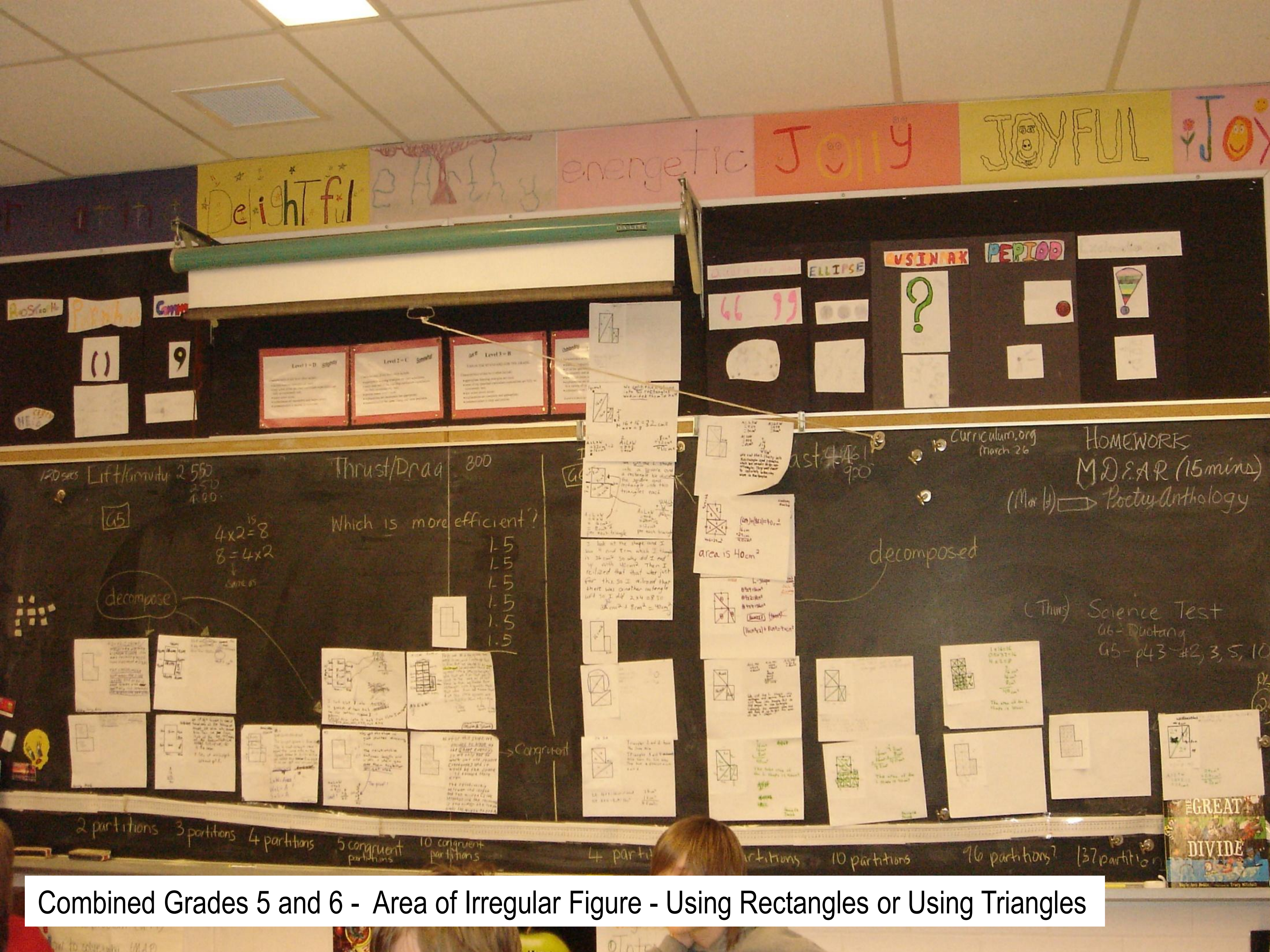

Combined Grades 5 and 6 - Area of Irregular Figure - Using Rectangles or Using Triangles

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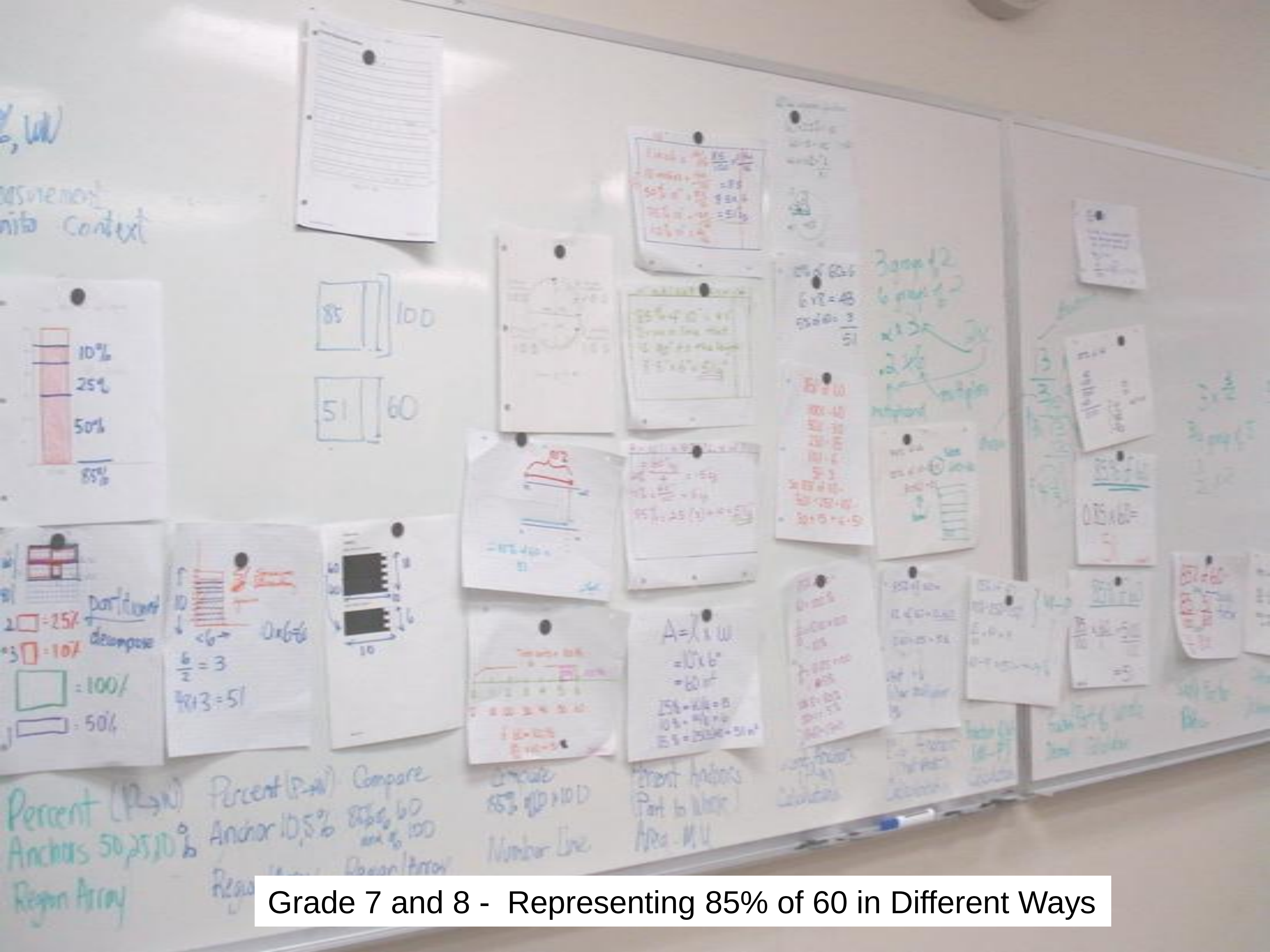


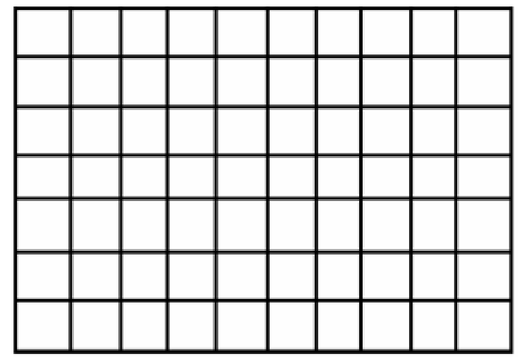
Grade 7 and 8 - Representing 85% of 60 in Different Ways

Three-Part Problem Solving Lesson Design

Before (Activating Knowledge)

What is similar and different about these numbers?

1, 64, 16, 4, 36, 9, 25, 49



Explain your ideas using a rectangular grid.

Our Ideas: square numbers, odd and even numbers, pattern of 1^2, 2^2, 3^2, 4^2, Arrays of 2x2, 3x3, 4x4, etc., area of squares with 2 linear units by 2 linear units, 3 lu x 3 lu, 4 lu x 4 lu; use of a table to record the lengths and area on a table of values, graphing to see a quadratic relationship

What is the mathematical purpose of this activation task?

Lesson Learning Goal
- **Curriculum Expectations** (Math Concepts/Terms)

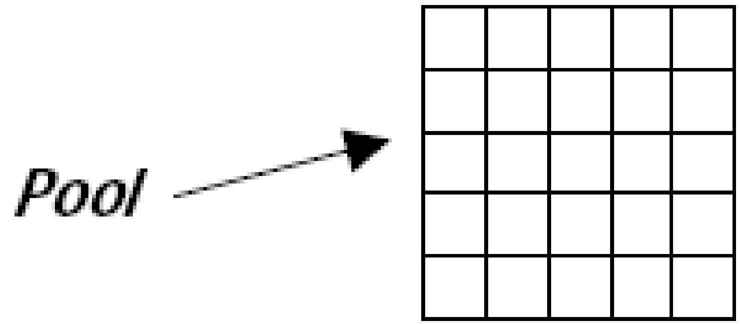
- Gr7 – model real life linear relationships graphically and algebraically
- Gr8 – model linear relationships using table of values, graphs, and equations, using a variety of tools
- Gr9 – construct tables of values, graphs, and equations, using a variety of tools to represent linear relations derived from descriptions of realistic situations
- Gr10 – construct tables of values and graphs, using a variety of tools to represent linear relations derived from the descriptions of realistic situations

Need to look at other grade specific expectations to determine the qualitative differences between these expectations ... Knowledge package, landscape, and learning trajectory

Three-Part Problem Solving Lesson Design

During (Working On It) – Pool Border Problem

1a. How many one-by-one tiles are required to surround a 5x5 pool?

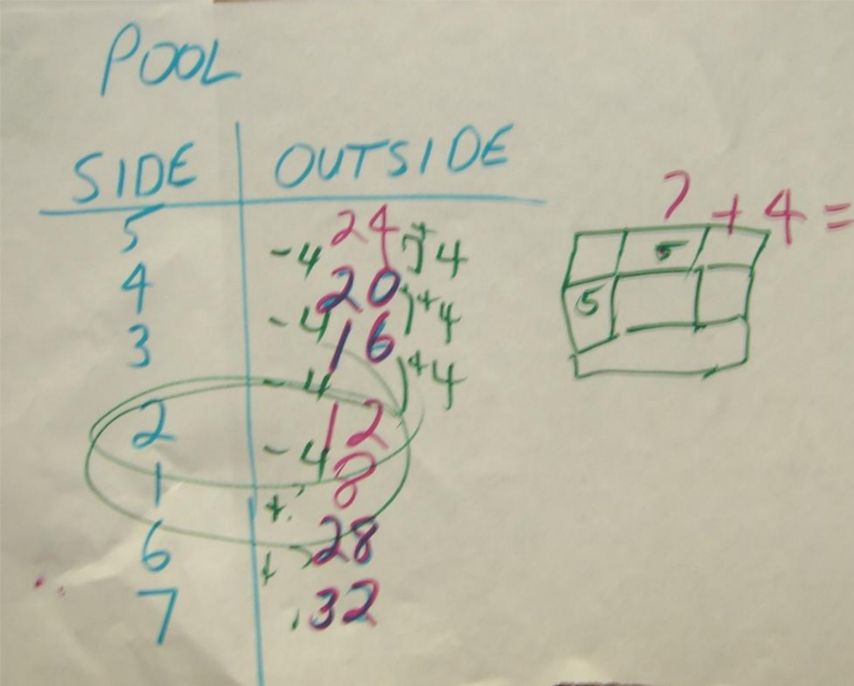


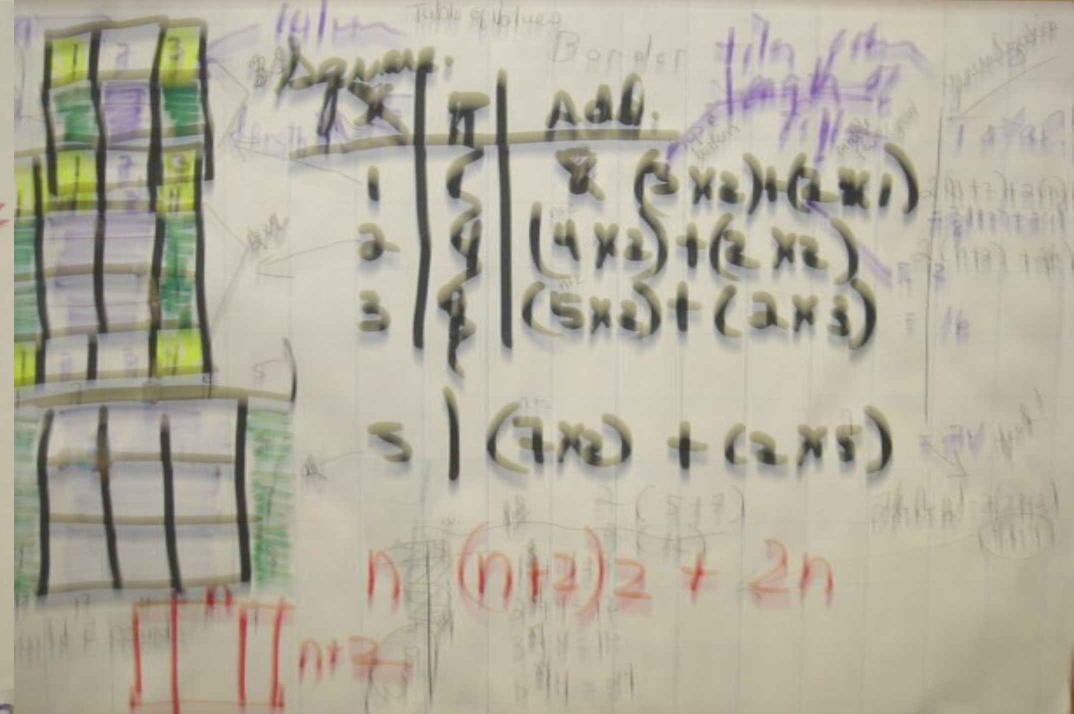
1b. Develop a generalization that predicts the number of tiles required to surround a square pool of any size.

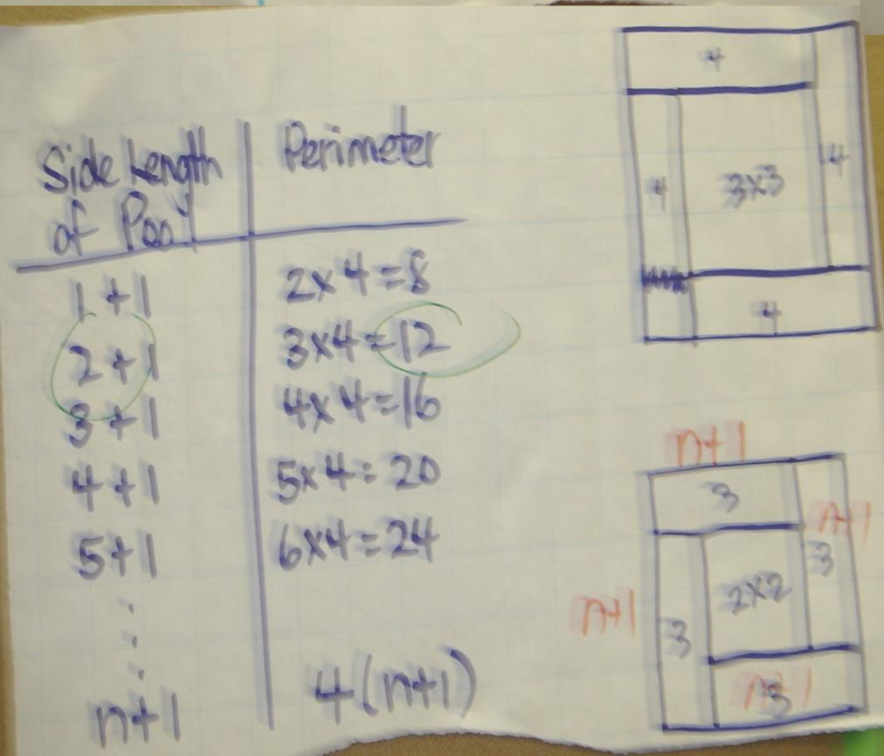
1c. Explain how your generalization relates to the size of the pool and the number of border tiles.

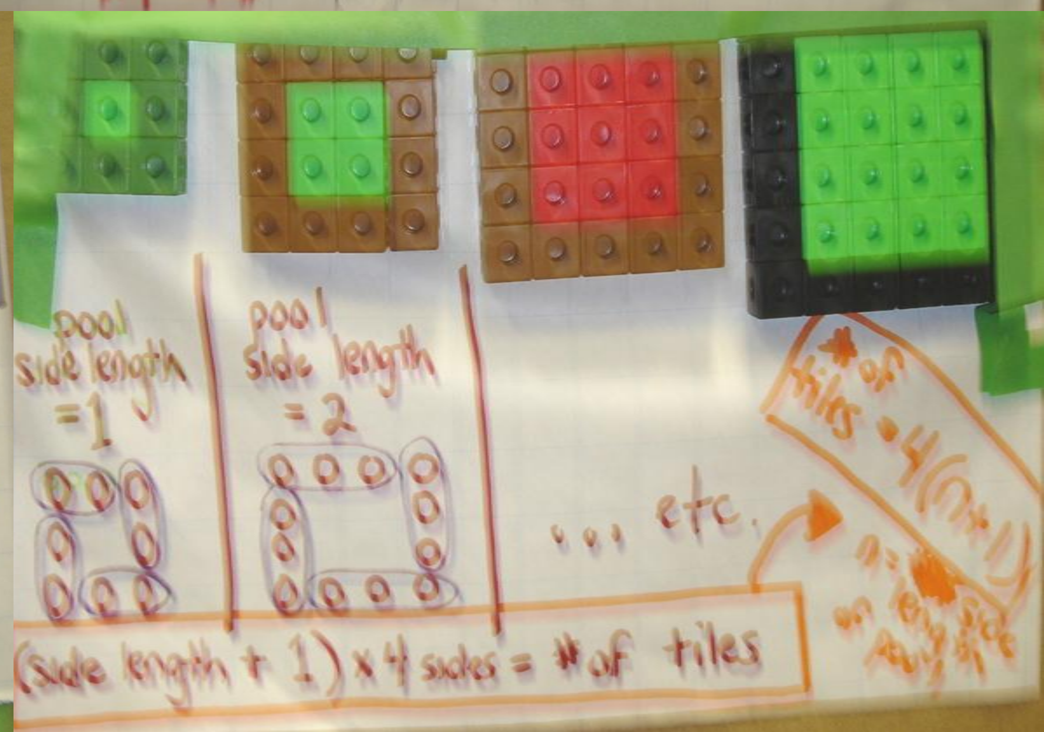
List grade- specific curriculum expectations that can be learned through this problem

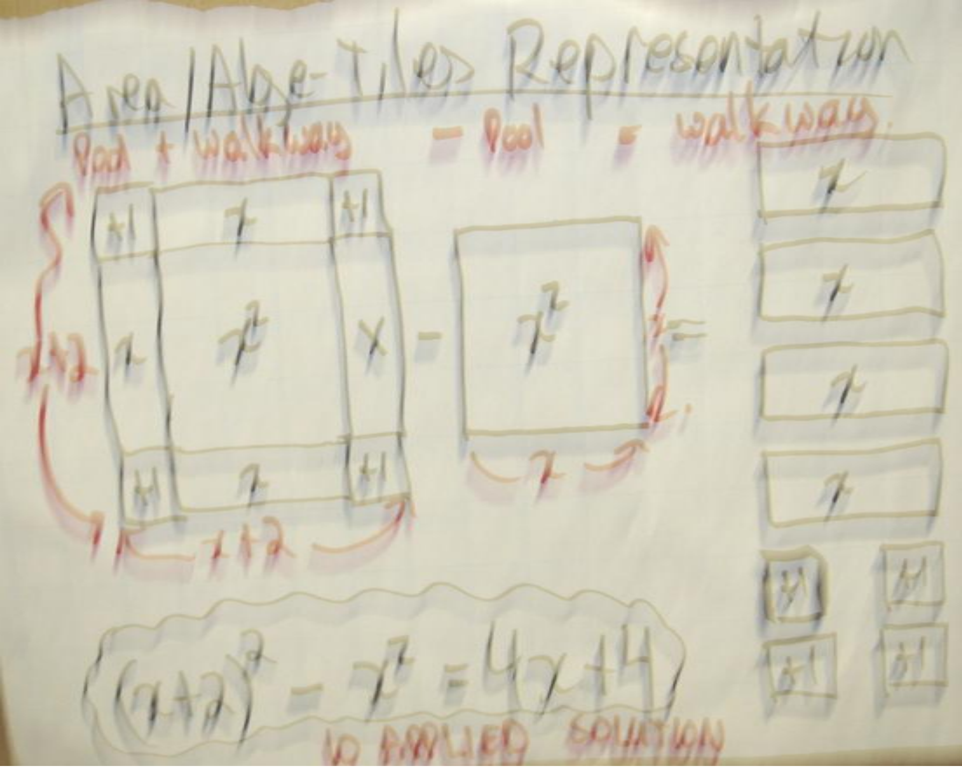
List the math concepts, strategies, and terms that you want to make explicit?


POOL
SIDE | OUTSIDE
5 | 24
4 | 20
3 | 16
2 | 12
1 | 8
6 | 28
7 | 32
7 + 4 =

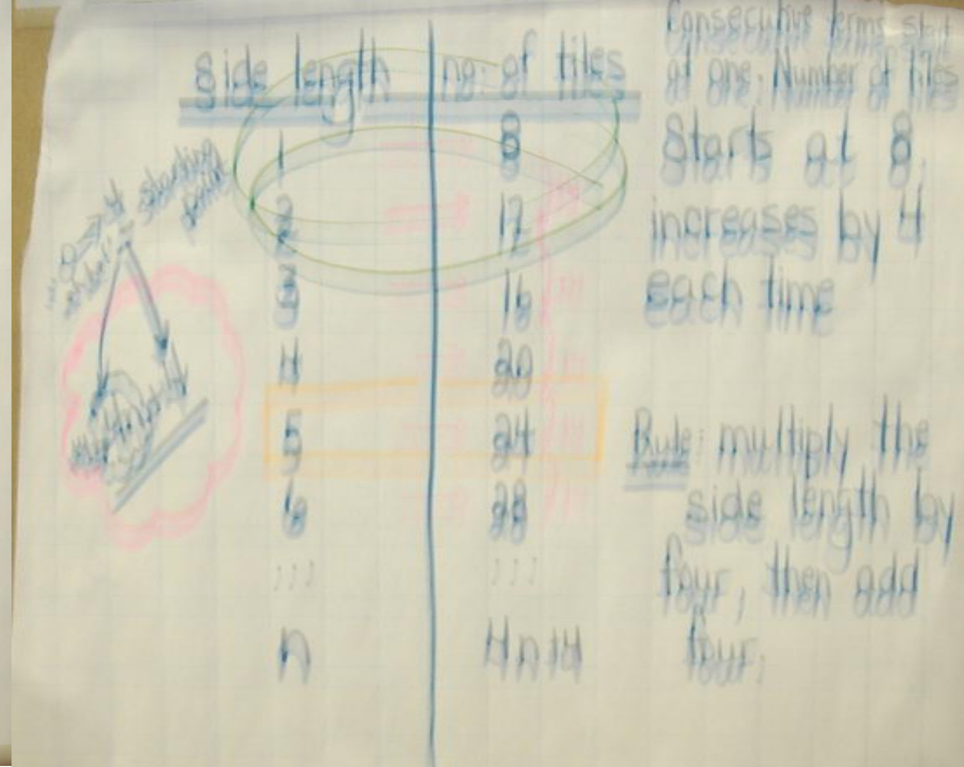

Square
(4 x2)+(2 x2)
(5x2)+(2x3)
(7x2) + (2x5)


Side Length of Pool | Perimeter
1+1 | 2x4=8
2+1 | 3x4=12
3+1 | 4x4=16
4+1 | 5x4=20
5+1 | 6x4=24
n+1 | 4(n+1)
3x3
2x2
n+1

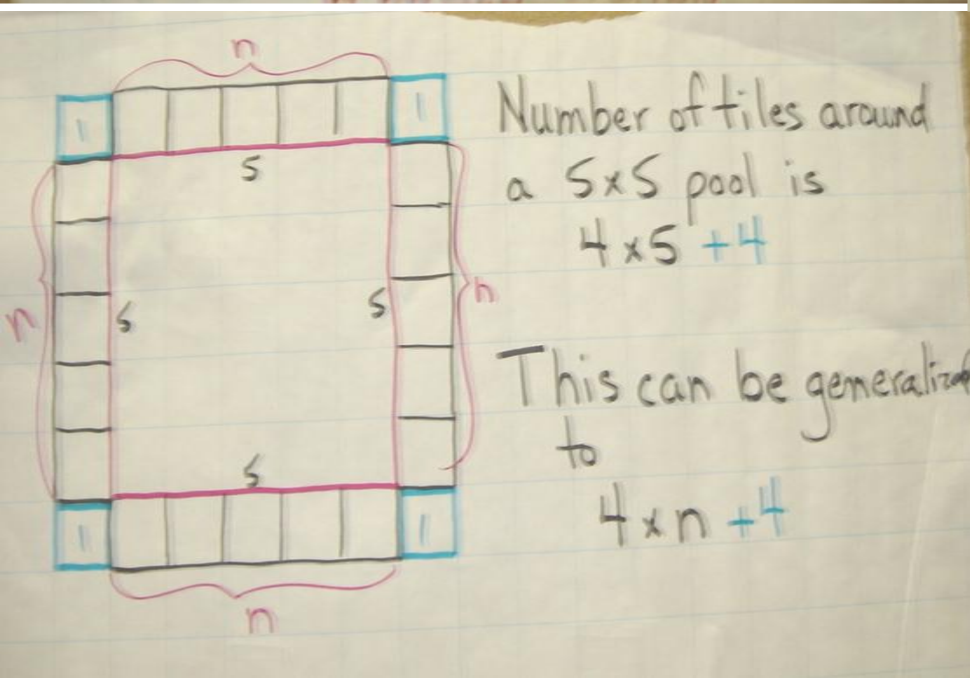

pool side length =1
pool side length =2
... etc.
(side length + 1) x 4 sides = # of tiles
# of tiles = 4(n+1)



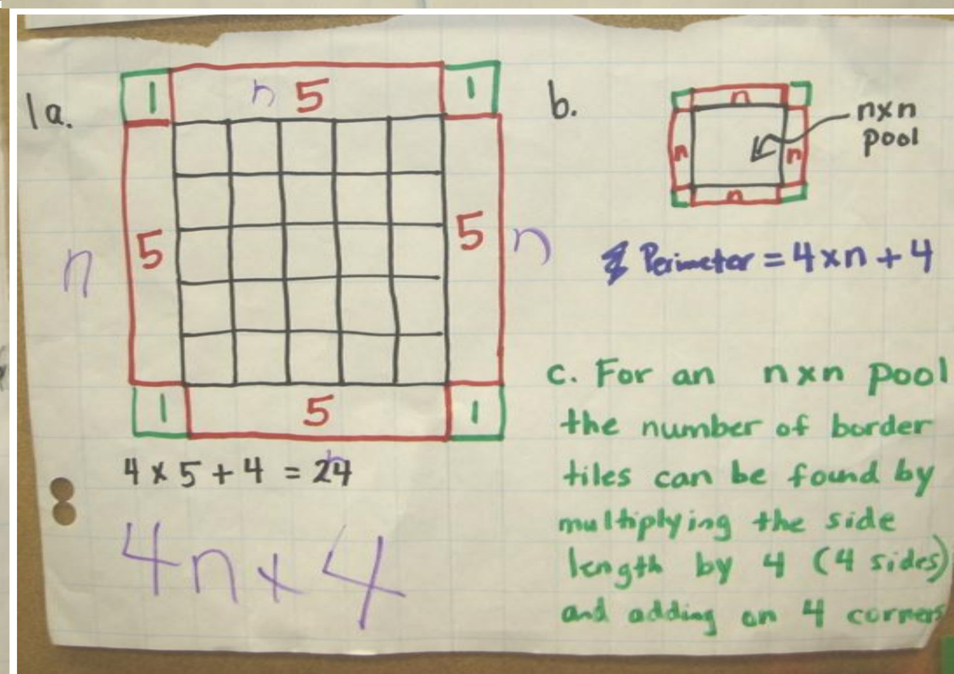
Area/Alge-Tiles Representation
Pool + walkway − Pool = walkway
x+2
x+2
$(x+2)^2 - x^2 = 4x+4$



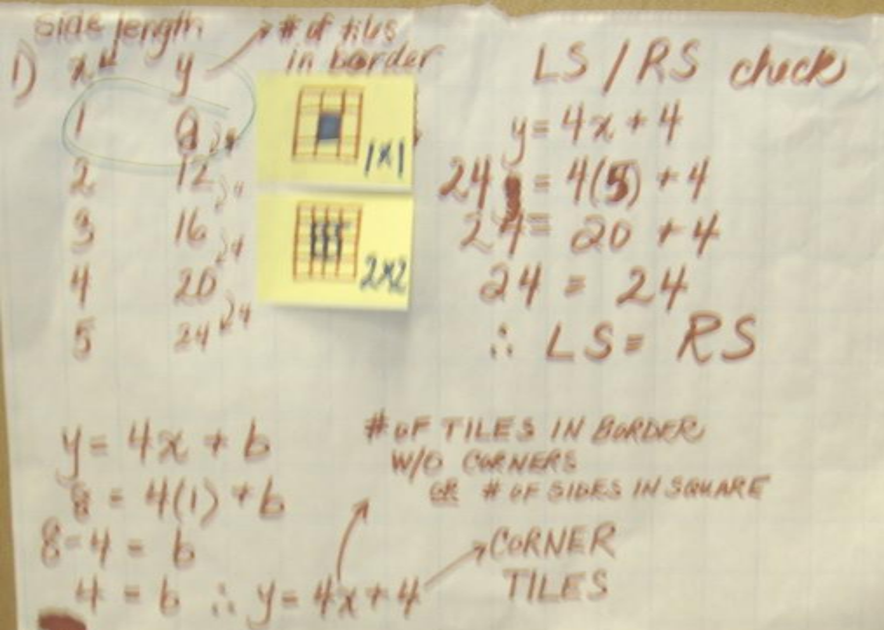
side length
no. of tiles
1 8
2 12
3 16
4 20
5 24
6 28
n 4n+4
Starts at 8,
increases by 4
each time
Rule: multiply the
side length by
four, then add
four.

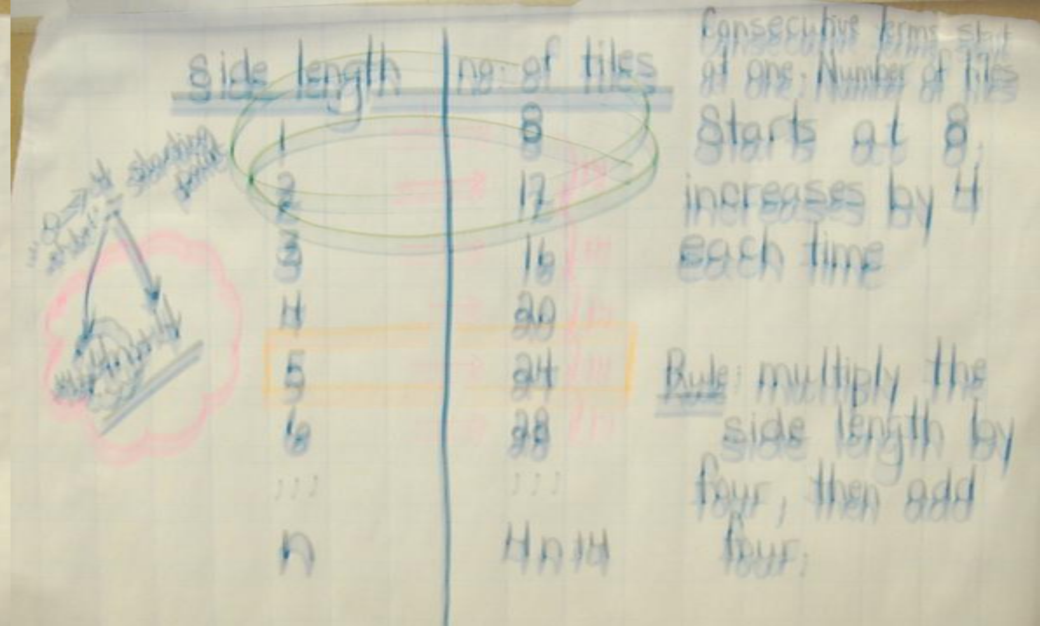


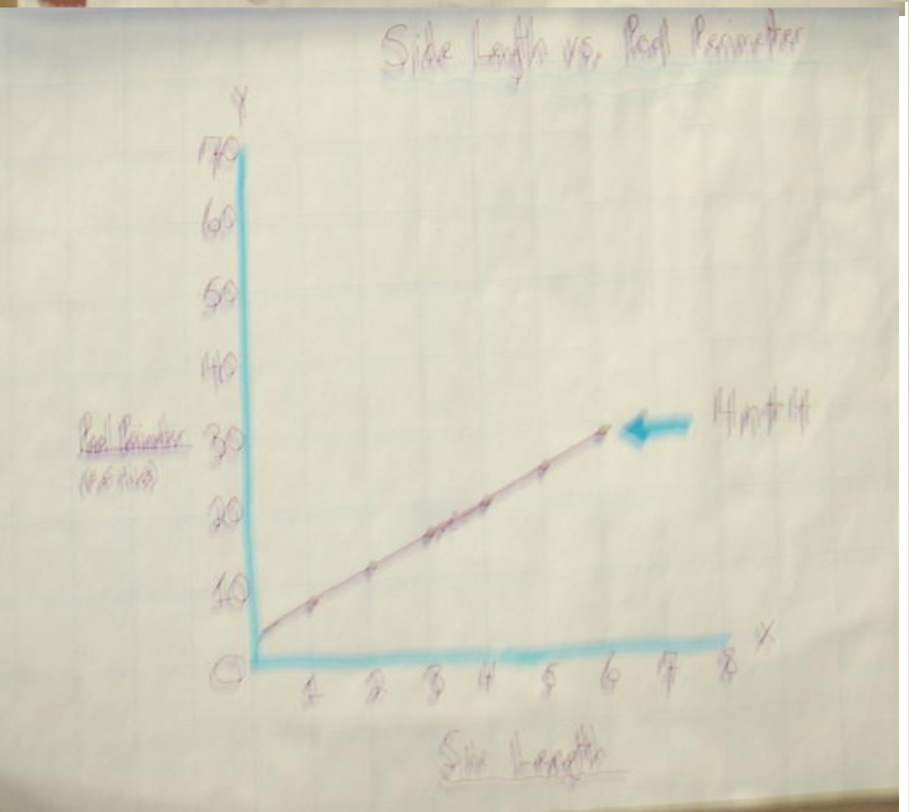
Number of tiles around
a 5×5 pool is
4×5 +4
This can be generalized
to
4×n +4

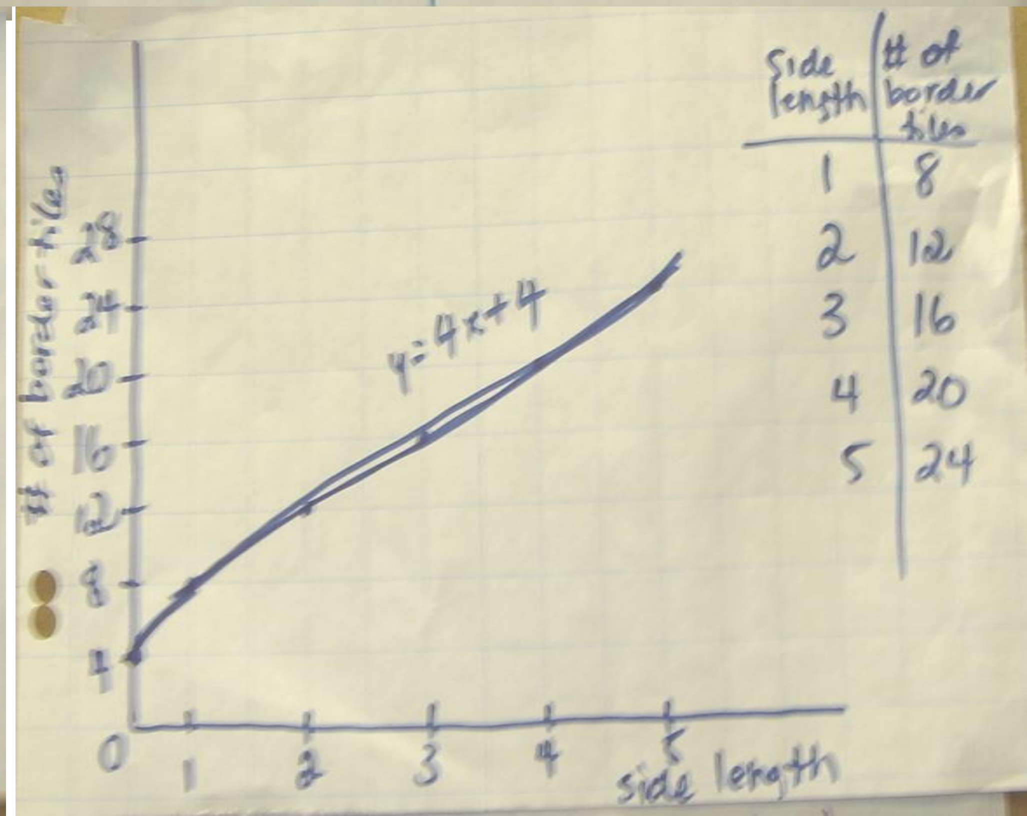


1a.
4×5+4 = 24
4n+4
b.
n×n
pool
Perimeter = 4×n+4
c. For an n×n pool
the number of border
tiles can be found by
multiplying the side
length by 4 (4 sides)
and adding on 4 corners


side length
# of tiles in border
1) x y
1 8
2 12
3 16
4 20
5 24
1×1
2×2
LS / RS check
y = 4x + 4
24 = 4(5) + 4
24 = 20 + 4
24 = 24
∴ LS = RS
y = 4x + b
8 = 4(1) + b
8 − 4 = b
4 = b ∴ y = 4x + 4
# OF TILES IN BORDER W/O CORNERS
OR # OF SIDES IN SQUARE
CORNER TILES

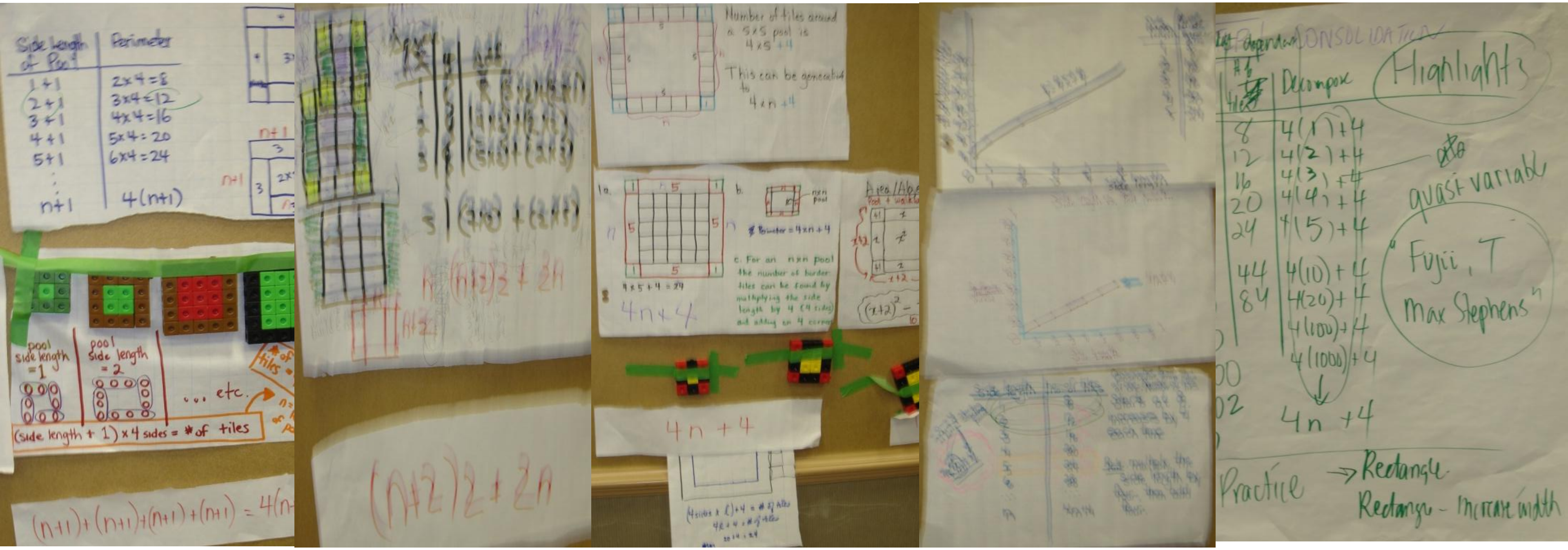

Side length
no. of tiles
1 8
2 12
3 16
4 20
5 24
6 28
n 4n+4
Consecutive terms start at one. Number of tiles
Starts at 8, increases by 4 each time
Rule: multiply the side length by four, then add four.


Side Length vs. Pool Perimeter
y
x
Pool Perimeter
Side Length
4n+4
0
10
20
30
40
50
60
70
1
2
3
4
5
6
7
8


side length
# of border tiles
1 8
2 12
3 16
4 20
5 24
y = 4x + 4
# of border tiles
side length
0
4
8
12
16
20
24
28
1
2
3
4
5

Pool Border Problem – Partial Bansho

(includes Before and During)

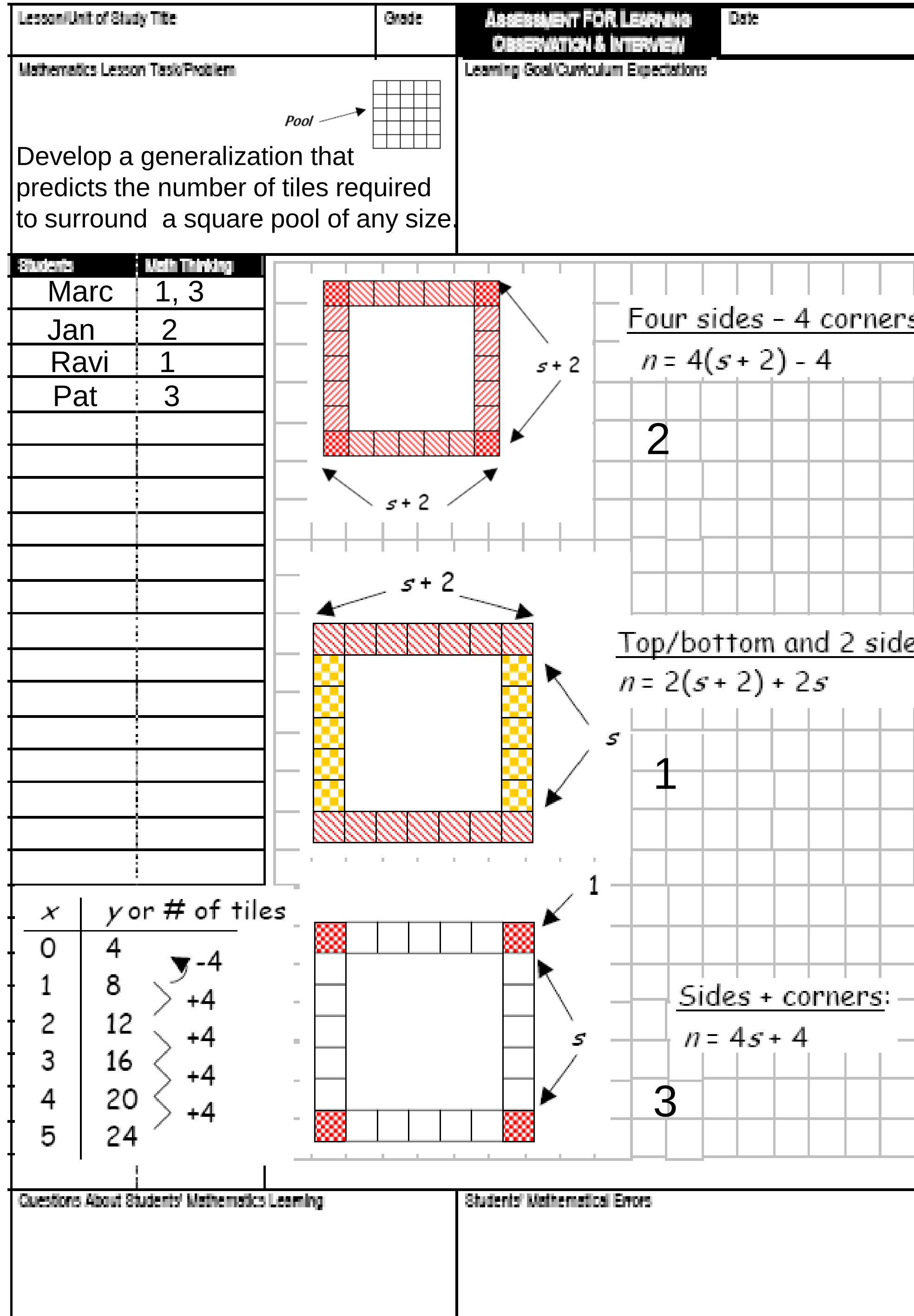


Solutions are organized by mathematical strategy (composition of parts of the pool Border, showing different algebraic expressions (akin to simplifying expressions to the Same expression 4(n+4); Highlights focus on the use of the data organized in a table of values (i.e., numeric cases), to build an expression showing constant and variable (in this case, the "quasi-variable").

Three-Part Problem Solving Lesson Design

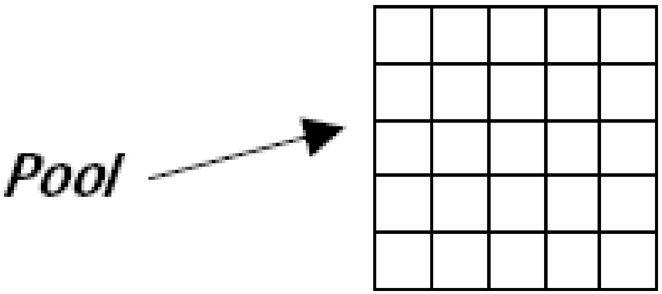
After (Consolidation) – (Ontario) Bansho

- What mathematics criteria will you use to organize the student solutions?
- How does the mathematics sorting criteria relate to the lesson learning goal?



Three-Part Problem Solving Lesson Design
After (Practice) –

- What problem(s) should we provide to enable students to practise (in pairs, individually) what they have learned?
- What learning goals (curriculum expectations) does your Practice Problem?



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1b. Develop a generalization that predicts the number of tiles required to surround a square pool of any size.

1c. Explain how your generalization relates to the size of the pool and the number of border tiles.

Three-Part Problem Solving Lesson Design

After (Practise)

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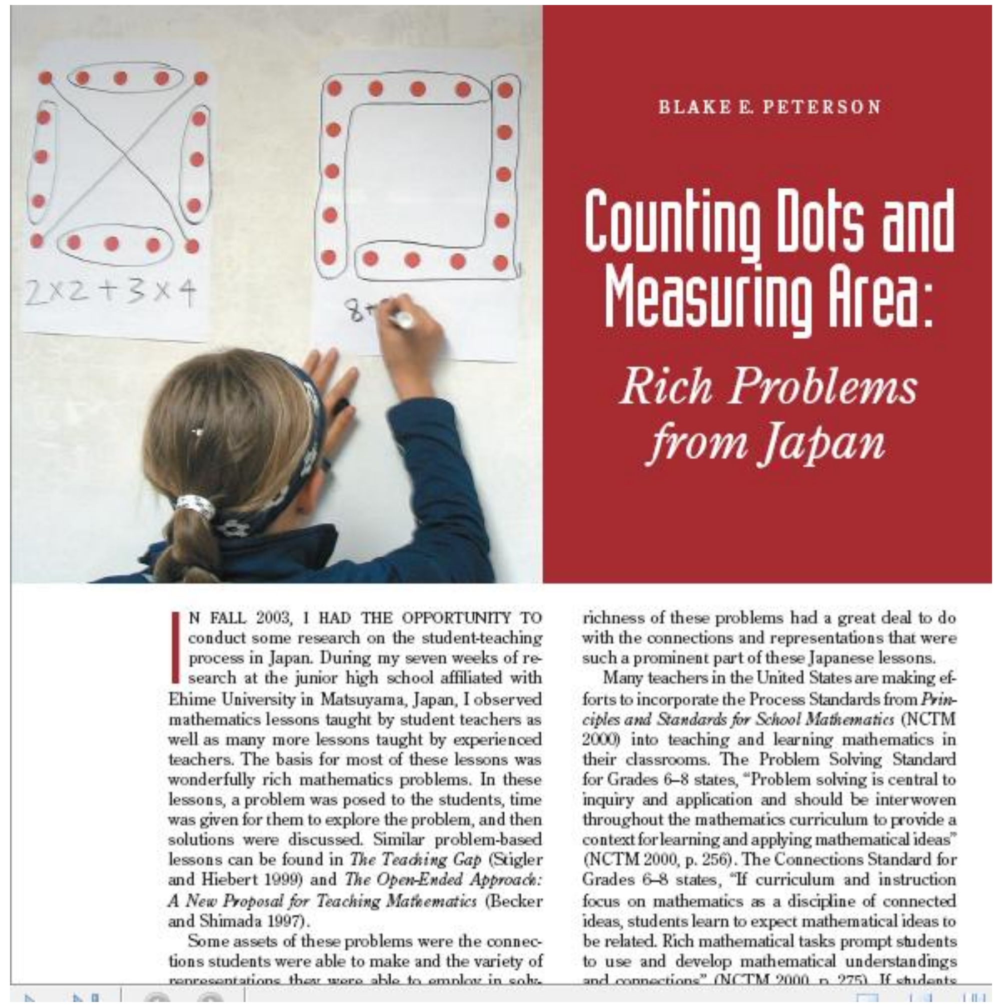
- What learning goals (curriculum expectations) does your Practice Problem?



BLAKE E. PETERSON

Counting Dots and Measuring Area: *Rich Problems from Japan*

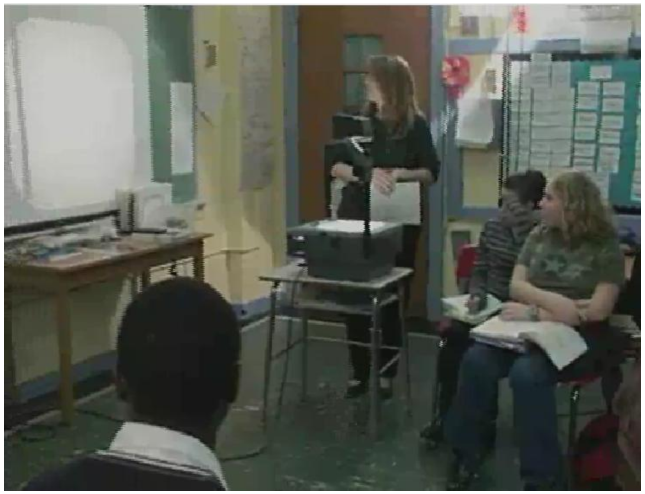
IN FALL 2003, I HAD THE OPPORTUNITY TO conduct some research on the student-teaching process in Japan. During my seven weeks of research at the junior high school affiliated with Ehime University in Matsuyama, Japan, I observed mathematics lessons taught by student teachers as well as many more lessons taught by experienced teachers. The basis for most of these lessons was wonderfully rich mathematics problems. In these lessons, a problem was posed to the students, time was given for them to explore the problem, and then solutions were discussed. Similar problem-based lessons can be found in *The Teaching Gap* (Stigler and Hiebert 1999) and *The Open-Ended Approach: A New Proposal for Teaching Mathematics* (Becker and Shimada 1997).

Some assets of these problems were the connections students were able to make and the variety of

richness of these problems had a great deal to do with the connections and representations that were such a prominent part of these Japanese lessons.

Many teachers in the United States are making efforts to incorporate the Process Standards from *Principles and Standards for School Mathematics* (NCTM 2000) into teaching and learning mathematics in their classrooms. The Problem Solving Standard for Grades 6–8 states, "Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas" (NCTM 2000, p. 256). The Connections Standard for Grades 6–8 states, "If curriculum and instruction focus on mathematics as a discipline of connected ideas, students learn to expect mathematical ideas to be related. Rich mathematical tasks prompt students to use and develop mathematical understandings

Other Lesson Approaches to the Pool Border Problem ... What Do You Think?

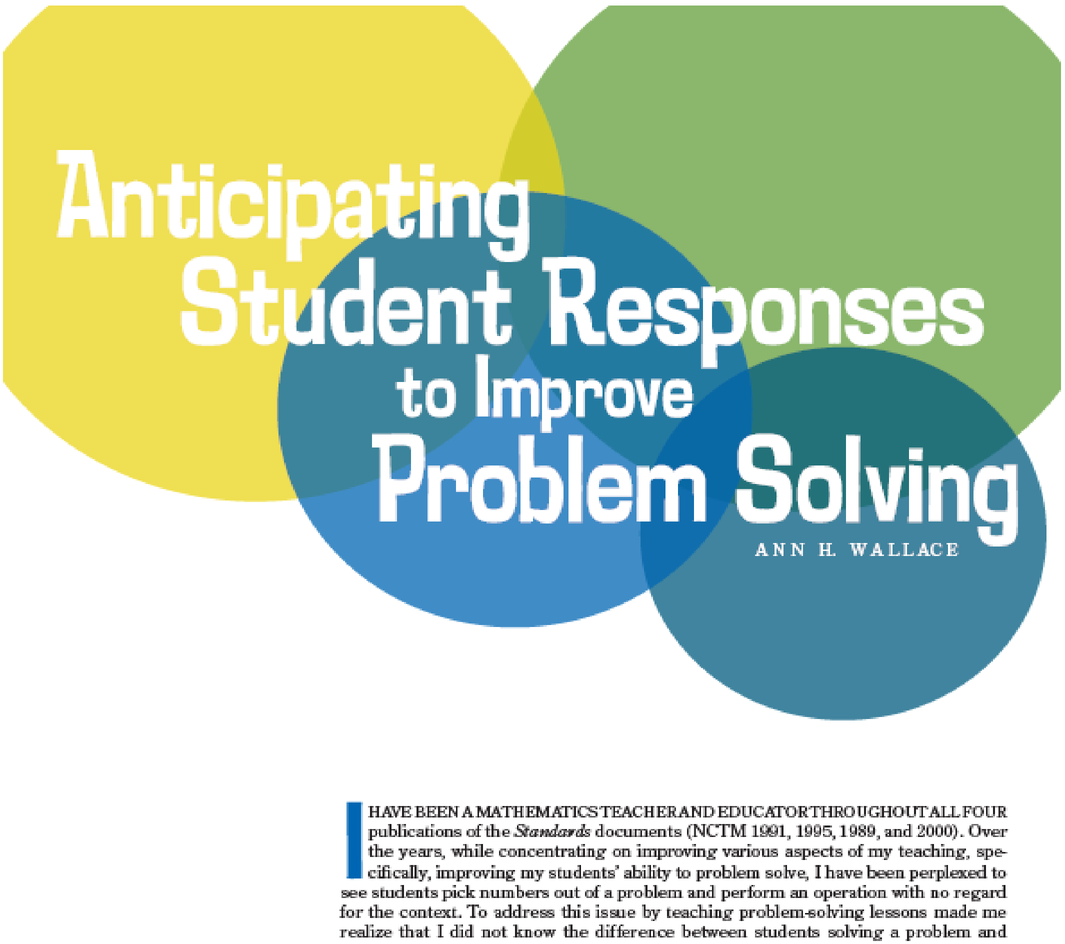


Criteria for Lesson Analysis for Teaching Through Problem Solving

- Before? During? After?
- Math Content coherence?
- Making Mathematical Connections?
- Who does the Mathematics Work?
- Nature of Mathematics Learning?
- Nature of the Mathematical Work?
- Kind of Mathematics Work Expected?
- Content Elaboration by Whom?

GAINS

Developing a Math Talk Community
- Anticipating Student Responses to Improve ...


Anticipating Student Responses to Improve Problem Solving

ANN H. WALLACE

I HAVE BEEN A MATHEMATICS TEACHER AND EDUCATOR THROUGHOUT ALL FOUR publications of the *Standards* documents (NCTM 1991, 1995, 1989, and 2000). Over the years, while concentrating on improving various aspects of my teaching, specifically, improving my students' ability to problem solve, I have been perplexed to see students pick numbers out of a problem and perform an operation with no regard for the context. To address this issue by teaching problem-solving lessons made me realize that I did not know the difference between students solving a problem and

Read 1 page ...
Key idea or strategy that supports the development of a math-talk community.

Linking to Differentiating Instruction

- Teach to the group but differentiate consolidation – open-routed problems provoke differentiated responses
- Teach different things to different groups – they listen in relation to their mathematical readiness, curiosity, and confidence
- Provide individual learning packages as much as possible - Consolidation
- Personalize both instruction and assessment – who is included and not included; what evidence of learning have you gathered from the classroom collective and individually
- Key concepts, choice, prior assessment

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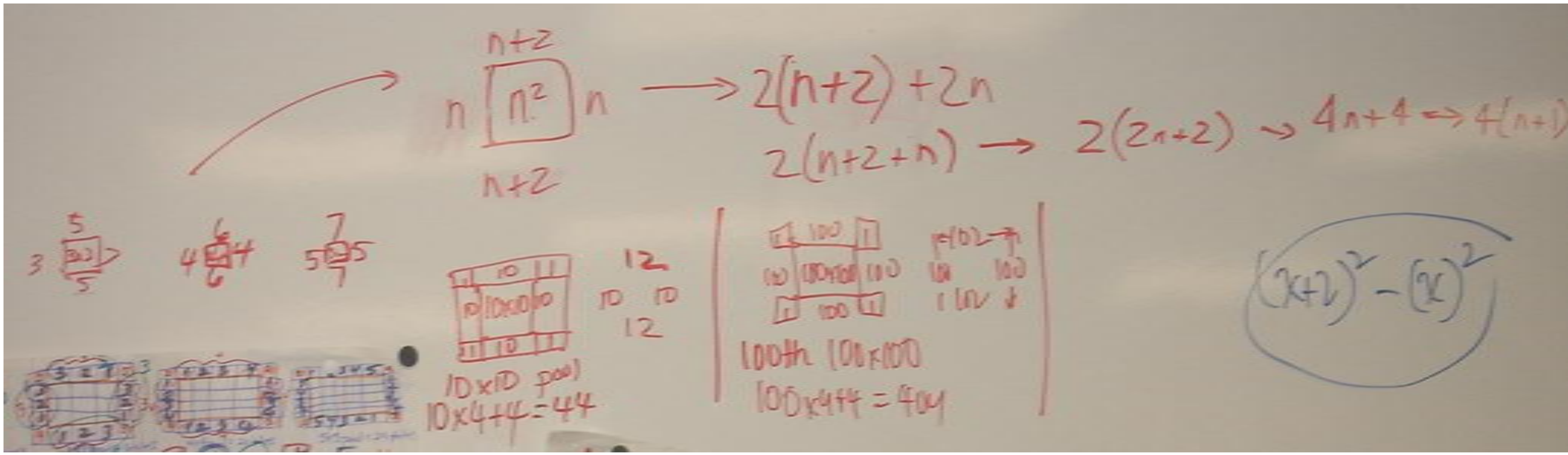
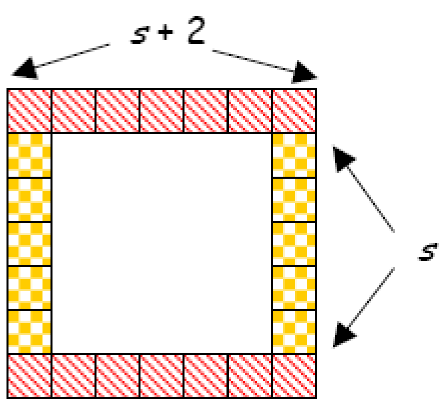
Three-Part Problem Solving Lesson Design
- Before (Comparing 2 Solutions)



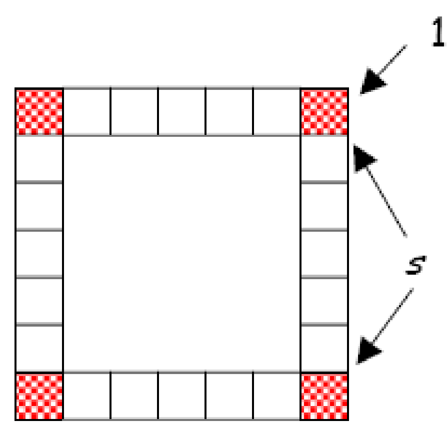
2. Top/bottom and 2 sides:

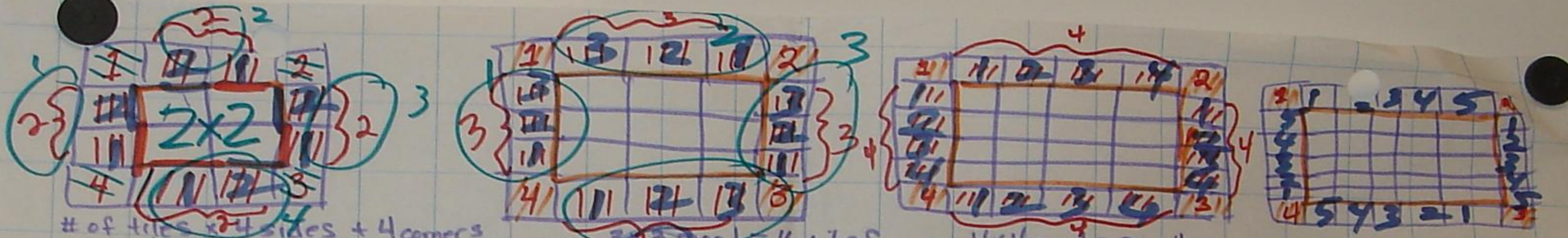
$n = 2(s + 2) + 2s$



1. Sides + corners:

$n = 4s + 4$





# of tiles x 4 sides + 4 corners

2 x 4 + 4 = 12

2x2 pool = 12 tiles

2+2+2+2 +4 = 12

4 x 2 + 4 = 12

3 x 3 + 4 ← Multiplicand ↖ Multiplier

4 x 4 + 4 Constant

5 x 5 + 4

X = w = l (x)	(y) COUNTED # of square tiles around pool	Decompose	Expression · 4x + 4 = y		(x, y)
same 2 same	12	4 + 4 + 4	4(2) + 4	4(2) + 4 = 12	(2, 12)
3	16	4 + 4 + 4 + 4	4(3) + 4	4(3) + 4 = 16	(3, 16)
4	20	4 + 4 + 4 + 4 + 4	4(4) + 4	4(4) + 4 = 20	(4, 20)
5	24	4 + 4 + 4 + 4 + 4 + 4	4(5) + 4	4(5) + 4 = 24	(5, 24)
N				4N + 4	

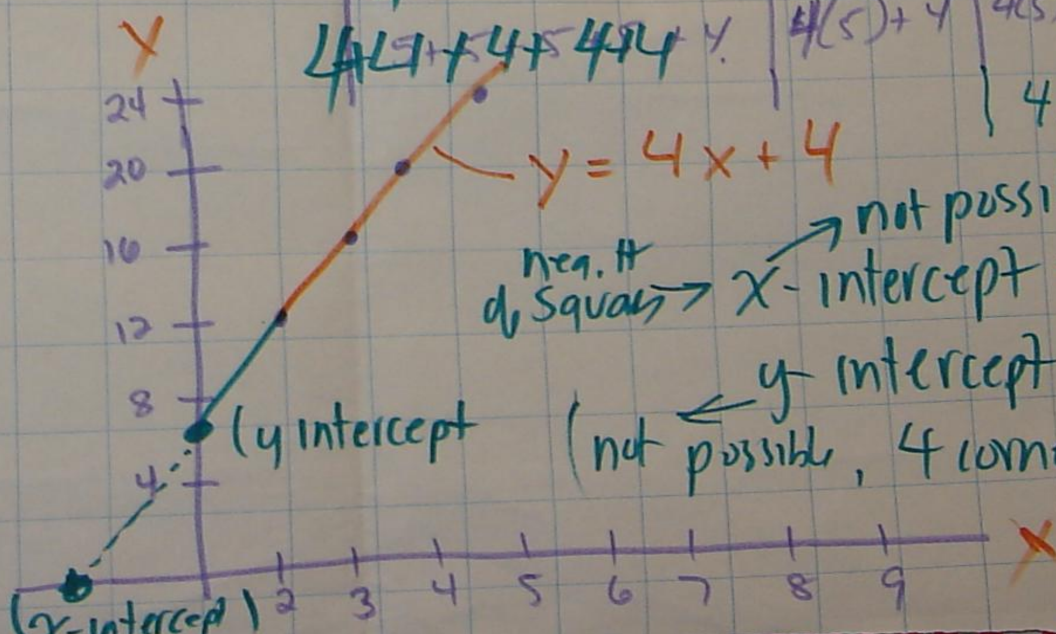
1 tile

# of square units

multiplicand (size of group)

multiplier (# groups)

variable



neg. # of squares → x-intercept = (, 0) → not possible

y intercept = (0,) ← (not possible, 4 corners)

Three-Part Problem Solving Lesson Design - During – Prom Dress and Gum Problem

Veronica and Caroline found the perfect prom dress which cost $80, but neither had enough money to buy it at the time.

Veronica put $20 aside that night and has been putting aside an additional $5 a day, since then.

Caroline put aside $8 every day since the day after she saw the dress. .

Heather says, “Wow! Caroline has more money saved.”

How many days has it been since Veronica and Caroline began saving?

Ken and his brother enjoy chewing gum. One day, the boys go to the candy store and buy several packages of gum.

Ken bought 18 ten-piece packages of gum, and his brother bought were five-piece packages of gum.

Every day, each of the boys finishes one whole pack of gum. One day, they looked at how much gum each boy had. Ken noticed that his brother had more pieces of gum than he had.

How many days has it been since the boys bought the gum?

Three-Part Problem Solving Lesson Design
- After (Consolidation)

- Show the curriculum expectations are the learning goal of your problem?
- Show the mathematics terms, symbols, key concepts, strategies are you making explicit to support the learning goal?
- Show the mathematics sorting criteria that you are using to show a recursive elaboration (transformation) across the solutions?
- Show the “Highlights” chart (matome) to make explicit how the different aspects of the student solutions contribute to the key mathematics that you want to be explicit .

Three-Part Problem Solving Lesson Design - **After (Practice)**

- What problem(s) should we provide to enable students to practise (in pairs, individually) what they have learned?
- What learning goals (curriculum expectations) does your Practice Problem?

GAINS

Looking Back
- Margaret Smith

- How are the problems similar? Different?
- How are the lesson designs similar? Different?

Beyond Presenting Good Problems
How a Japanese Teacher Implements a Mathematics Task

Margaret Smith

For several years teachers have been encouraged to use tasks that engage students in mathematical thought processes. For example, the recommendations by the National Council of Teachers of Mathematics (NCTM) in both the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and the *Principles and Standards for School Mathematics* (NCTM 2000) encourage teachers and curriculum developers to use problems that go beyond practicing routine procedures to problems that help students build mathematical connections and develop and apply mathematical concepts. The expectation is that, by incorporating problems that engage students in mathematical thought processes, teachers will provide students with opportunities to conjecture, reason, and develop new mathematical ideas. Engaging students in such discussions is relatively new for both teachers and students. To help teachers learn some useful strategies, this article offers examples and discussions of ways teachers can implement problems to help students engage in mathematical thought processes.

Tasks That Help Students Make Connections

The initial release of data from the Third International Mathematics and Science Video Study (TIMSS Video Study) promoted a lot of discussion

The data used in this article were generated as part of requirements for a doctoral dissertation at the University of Delaware using video data from the TIMSS Laboratory at the University of California, Los Angeles. I wish to thank James Hiebert for his wisdom and guidance in the project, as well as

"Ontario" Bansho
- as an Instructional Strategy

To provoke students' mathematical thinking to be explicit when solving problems through the organization and annotation of student work samples and classroom discourse

Preparation – Teacher:

- cleared black board space
- chose lesson problem based on grade-specific expectations
- solve the problem prior to the lesson to anticipate a range of solutions, to decide on the math tools students need, and to anticipate the key mathematical aspects of solutions
- prepare (if possible) mathematical labels for categories of the solutions
- provide square grid chart paper (in eighths), markers so that solutions are visible to students

Actions – Teacher:

- activate student knowledge and experience at start of lesson
- circulate among the students as they are solving a problem to discern the range of the solutions and the mathematical connections to prompt
- sort and classify their solutions (like a concrete graph) according to mathematical criteria related to the learning goal(s) of lesson
- select which students will share their solutions 1st, 2nd, 3rd, so to build mathematics knowledge through analysis and discussion

Grade 3 - Dividing 12 by 5

"Ontario" Bansho - As a Classroom Artefact

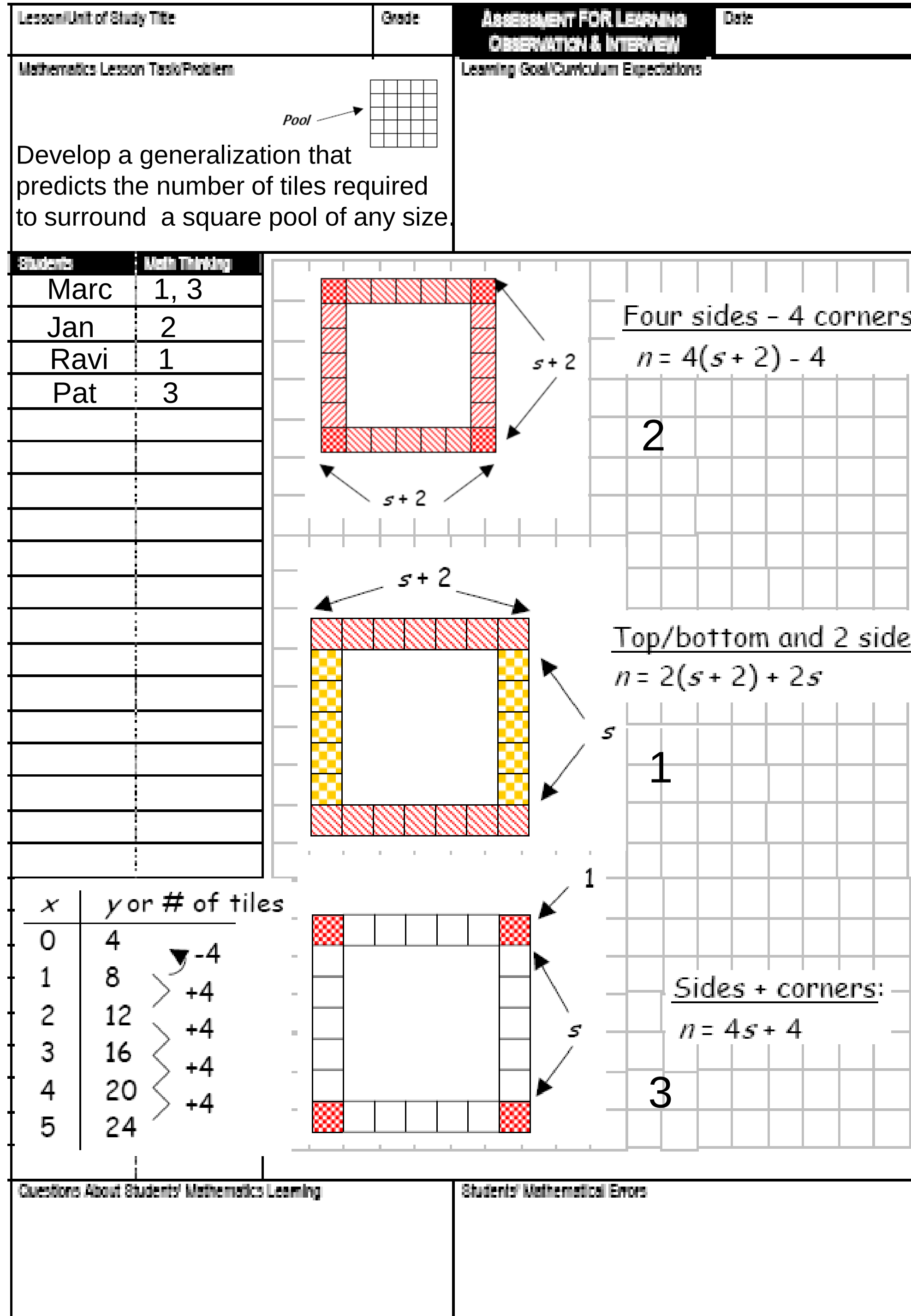
An artefact produced collectively by students and teacher that publicly displays the mathematical relationship among students' solutions; could be used as a mathematics landscape for learning or as a mathematics anchor chart.

- The flow of three-part lesson (e.g., problems, key words, strategies) are posted on the blackboard, from left to right
- Student solutions to lesson problem are categorized and labelled like a concrete graph (from 4 solutions to all solutions)
- Student solutions are mathematically annotated to highlight key concepts, representations, strategies, vocabulary, and symbols
- Student work samples and ideas from previous lessons are posted to revisit prior mathematical ideas and strategies

Grade 2 - Addition Strategies

"Ontario Bansho" as Assessment for Learning

- To discern the range and the mathematical relationships between ideas, strategies, and models of representation used to solve a problem by students.

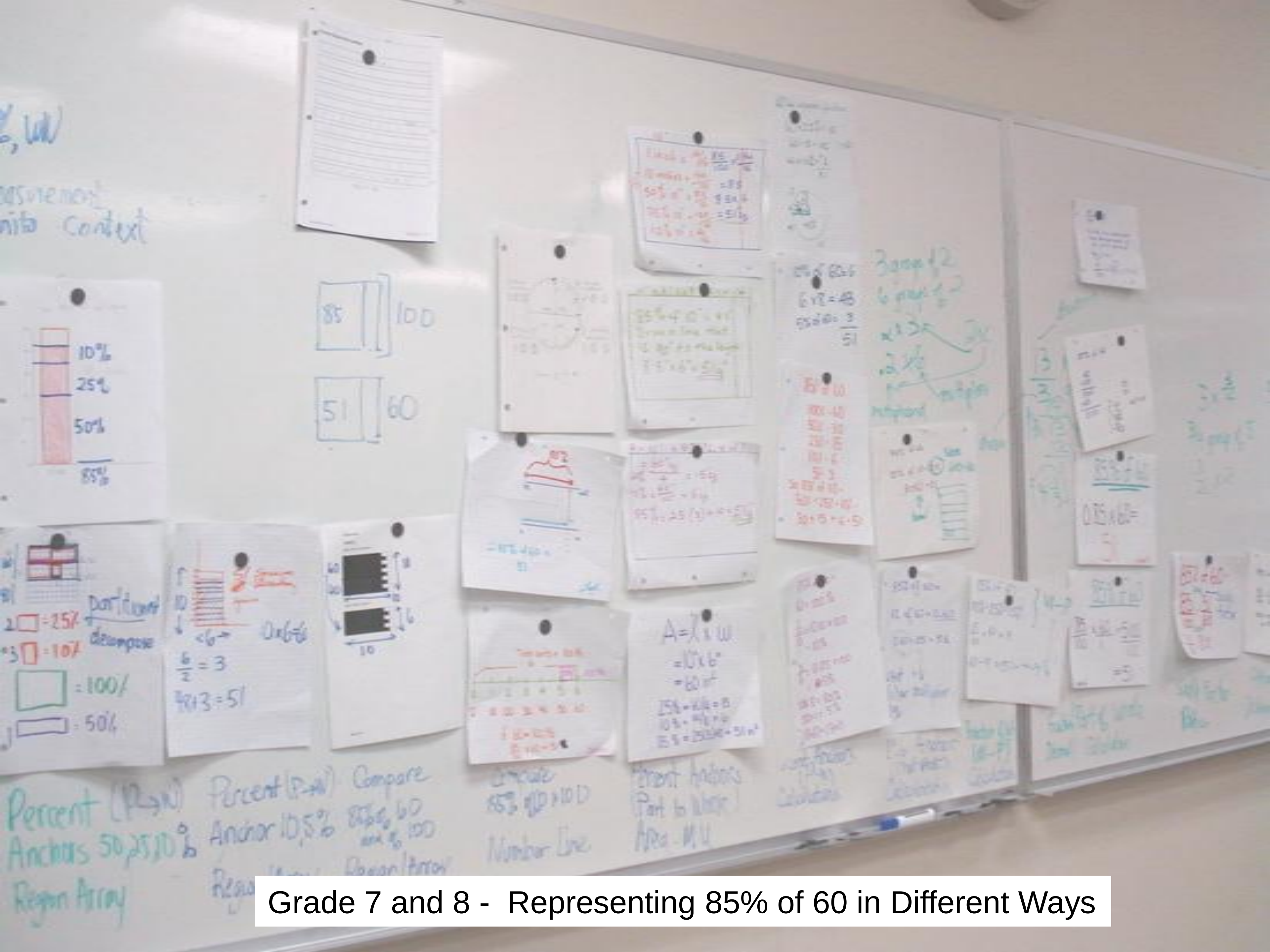


Grade 6 - Surface Area of Triangular Prisms

"Ontario" Bansho as a Mathematics Professional Learning Strategy

It develops teacher's knowledge of mathematics *for teaching,* to construct a mathematics landscape evoked through the solving of problems

- In preparation for teaching mathematics, solve a mathematics problem in different ways with colleagues
- Organize the solutions to show the mathematical relationship between the solutions, often in a mathematical developmental continuum
- Use this artefact to construct a mathematics landscape (often across strands) to explain the range of mathematical ideas, strategies, and models used to solve a problem



Grade 7 and 8 - Representing 85% of 60 in Different Ways

Learn Mathematics *for* Teaching
(Ball, 2005)

- Figure out why procedures work, not just how to do them
- Try to solve problems in more than one way
- Listen to and probe others' thinking, especially when struggling
- Study students' thinking and work
- Talk in class; practise speaking mathematics

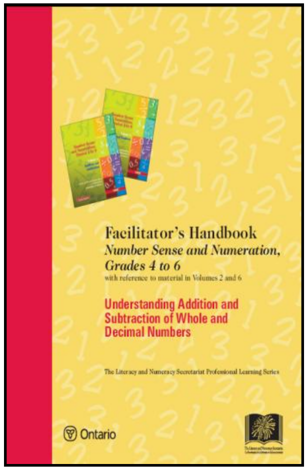
LNS Resources Highlighting Bansho

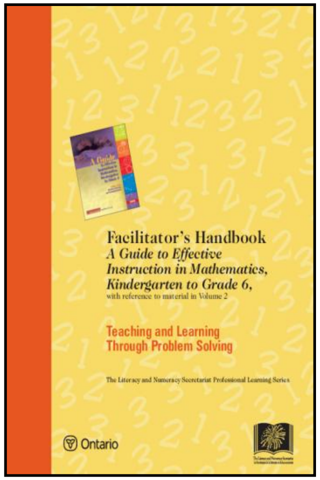
LNS Webcasts – www.curriculum.org

- March 2007 - Making Mathematics Accessible for all Students (Grades 1, 2/3, 4, 5/6)
- June 2007 - Coaching for Student Success (Gr3)
- March 2008 - Investigating High Yield Strategies (Gr6)

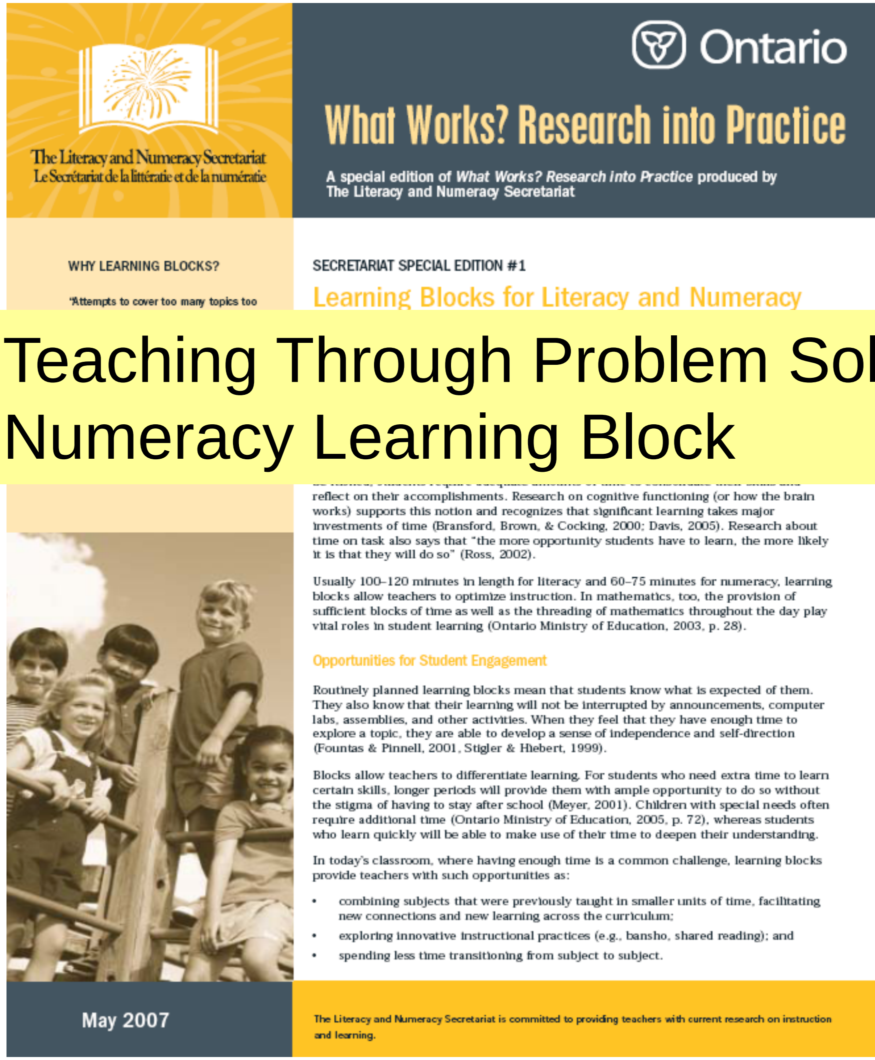
LNS Facilitator Handbooks at www.curriculum.org (Coaching Institute – Numeracy Resources)

- Addition and Subtraction
- Multiplication and Division
- Teaching Through Problem Solving





Other LNS Resources



Ontario

The Literacy and Numeracy Secretariat
Le Secrétariat de la littératie et de la numératie

What Works? Research into Practice

A special edition of *What Works? Research into Practice* produced by The Literacy and Numeracy Secretariat

WHY LEARNING BLOCKS?

"Attempts to cover too many topics too

SECRETARIAT SPECIAL EDITION #1

Learning Blocks for Literacy and Numeracy

reflect on their accomplishments. Research on cognitive functioning (or how the brain works) supports this notion and recognizes that significant learning takes major investments of time (Bransford, Brown, & Cocking, 2000; Davis, 2005). Research about time on task also says that "the more opportunity students have to learn, the more likely it is that they will do so" (Ross, 2002).

Usually 100–120 minutes in length for literacy and 60–75 minutes for numeracy, learning blocks allow teachers to optimize instruction. In mathematics, too, the provision of sufficient blocks of time as well as the threading of mathematics throughout the day play vital roles in student learning (Ontario Ministry of Education, 2003, p. 28).

Opportunities for Student Engagement

Routinely planned learning blocks mean that students know what is expected of them. They also know that their learning will not be interrupted by announcements, computer labs, assemblies, and other activities. When they feel that they have enough time to explore a topic, they are able to develop a sense of independence and self-direction (Fountas & Pinnell, 2001, Stigler & Hiebert, 1999).

Blocks allow teachers to differentiate learning. For students who need extra time to learn certain skills, longer periods will provide them with ample opportunity to do so without the stigma of having to stay after school (Meyer, 2001). Children with special needs often require additional time (Ontario Ministry of Education, 2005, p. 72), whereas students who learn quickly will be able to make use of their time to deepen their understanding.

In today's classroom, where having enough time is a common challenge, learning blocks provide teachers with such opportunities as:

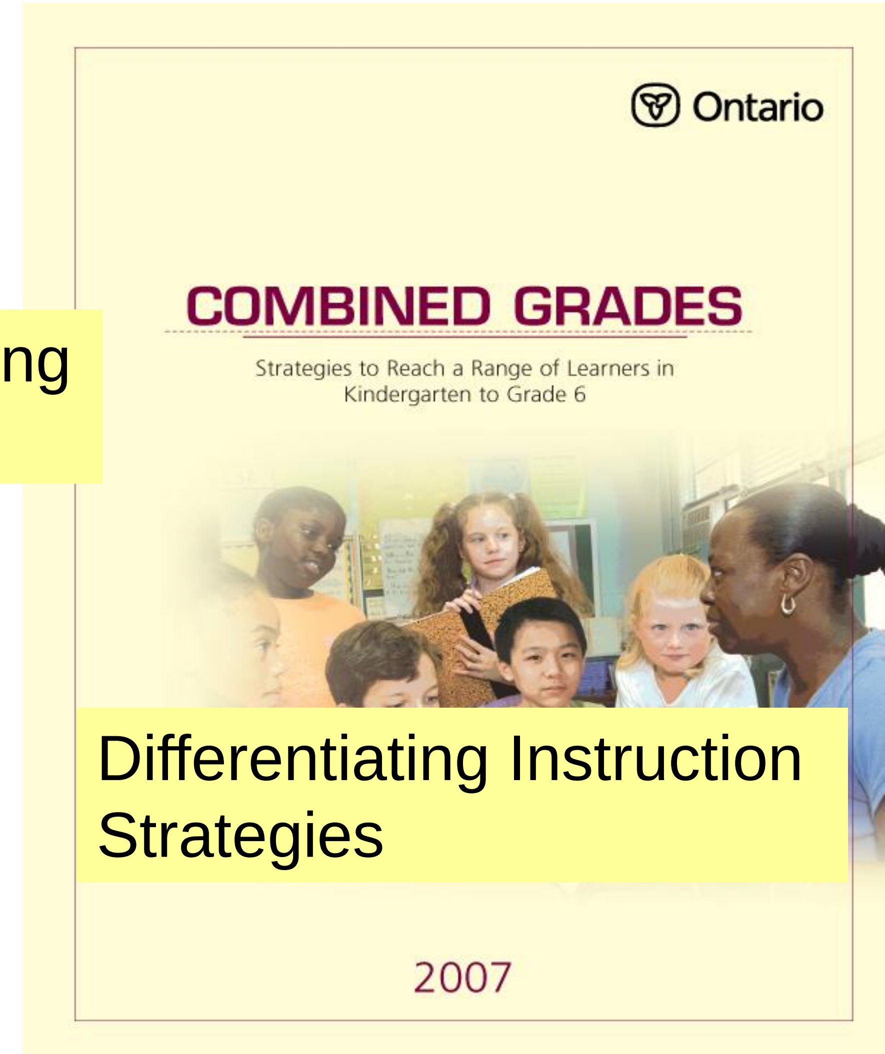
- combining subjects that were previously taught in smaller units of time, facilitating new connections and new learning across the curriculum;
- exploring innovative instructional practices (e.g., bansho, shared reading); and
- spending less time transitioning from subject to subject.

May 2007

The Literacy and Numeracy Secretariat is committed to providing teachers with current research on instruction and learning.

Teaching Through Problem Solving Numeracy Learning Block

http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/capacityBuilding.html



Differentiating Instruction Strategies

http://www.edu.gov.on.ca/eng/literacynumeracy/combined.html